10.3 Ellipses II

Monday, April 26, 2010
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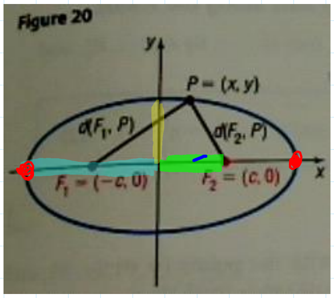
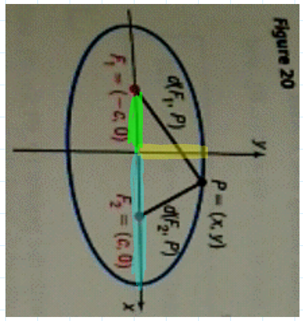
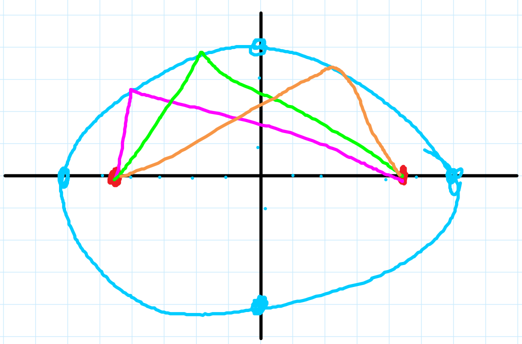




$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

$$\cancel{b^2=a^2-c^2} \quad \text{or} \quad c^2=a^2-b^2$$

$$\frac{x^2}{\underset{6^2=}{36}}+\frac{y^2}{\underset{4^2}{16}}=1$$

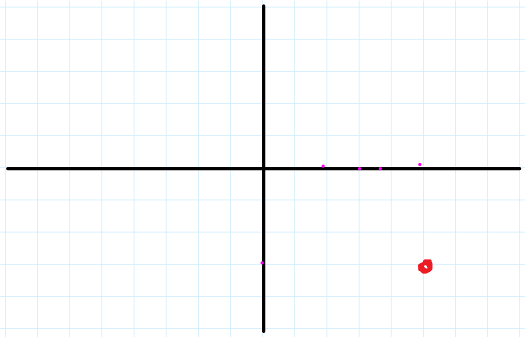


$$c^2 = 6^2 - 4^2$$

$$c^2 = 36 - 16 = 20$$

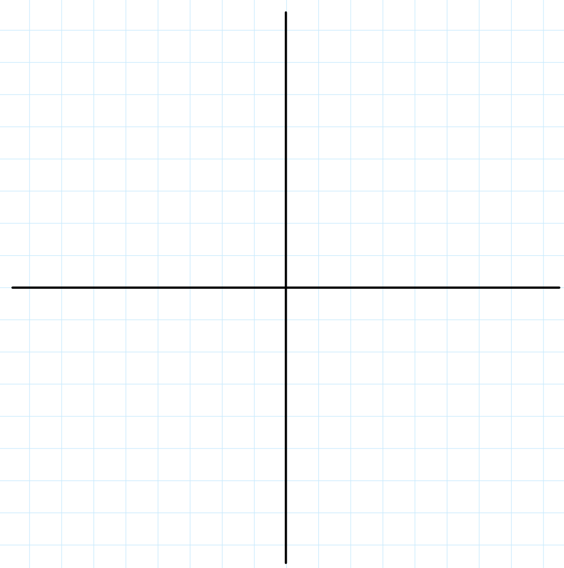
$$c = \sqrt{20} \approx 4.5$$

$$\frac{(x-5)^2}{36}+\frac{(y+3)^2}{16}=1$$



Analyze the ellipse:

$4x^2 + y^2 - 8x + 4y + 4 = 0$



$9x^2 + 4y^2 - 54x + 16y + 61 = 0$

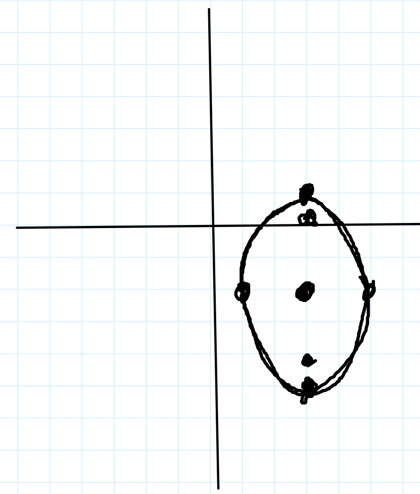
$(9x^2 - 54x + \_\_) + (4y^2 + 16y + \_\_) = -61$

$9(x^2 - 6x + \underline{9}) + 4(y^2 + 4y + \underline{4}) = -61 + 81 + 16$

$$\frac{9(x-3)^2}{36} + \frac{4(y+2)^2}{36} = \frac{36}{36}$$

$$\frac{(x-3)^2}{4} + \frac{(y+2)^2}{9} = 1$$

$$\frac{(x-3)^2}{2^2} + \frac{(y+2)^2}{3^2} = 1$$



Center = $(3, -2)$

$a = 3$

$b = 2$

$c = 2.2$

$c^2 = 9 - 4 = 5$

$c = \sqrt{5} \approx 2.2$

#48) $x^2 + 3y^2 - 12y + 9 = 0$

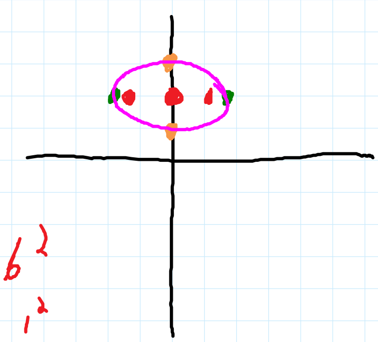
$$x^2 + (3y^2 - 12y \quad) = -9$$

$$x^2 + 3(y^2 - 4y + 4) = -9 + 12$$

$$\frac{x^2}{3} + \frac{3(y-2)^2}{3} = \frac{3}{3}$$

$$\frac{x^2}{3} + \frac{(y-2)^2}{1} = 1$$

$$\frac{(x-0)^2}{(\sqrt{3})^2} + \frac{(y-2)^2}{1^2} = 1$$



$\sqrt{3} \approx 1.7$

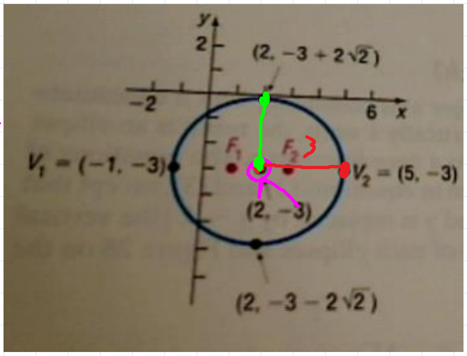
$c^2 = a^2 - b^2$

$c^2 = (\sqrt{3})^2 - 1^2$

$c^2 = 2$

$c = \sqrt{2} \approx 1.4$

Find the equation of the ellipse:



$$\frac{(x-2)^2}{3^2}+\frac{(y-3)^2}{(2\sqrt{2})^2}=1$$

$$\left(\frac{(x-2)^2}{9}+\frac{(y+3)^2}{8}=1\right)$$

$$(2\sqrt{2})^2=4\cdot 2=8$$

$$8(x-2)^2+9(y+3)^2=72$$

10.3: 13-17 all, 21, 27, 29,39, 45-49 odd, 55

In Problems 17–26, find the vertices and foci of each ellipse. Graph each equation by hand. Verify your graph using a graphing utility.

17. $\frac{x^2}{25}+\frac{y^2}{4}=1$ 18. $\frac{x^2}{9}+\frac{y^2}{4}=1$ 19. $\frac{x^2}{9}+\frac{y^2}{25}=1$ 20. $x^2+\frac{y^2}{16}=1$

21. $4x^2+y^2=16$ 22. $x^2+9y^2=18$ 23. $4y^2+x^2=8$ 24. $4y^2+9x^2=36$

25. $x^2+y^2=16$ 26. $x^2+y^2=4$

In Problems 27–38, find an equation for each ellipse. Graph the equation by hand.

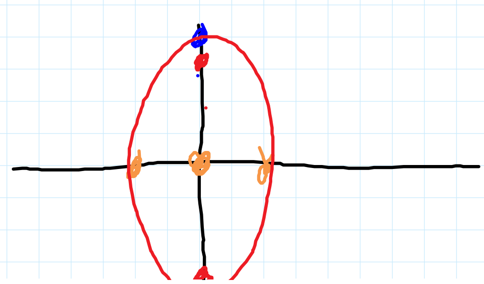
27. Center at (0, 0); focus at (3, 0); vertex at (5, 0)
28. Center at (0, 0); focus at (−1, 0); vertex at (3, 0)
29. Center at (0, 0); focus at (0, −4); vertex at (0, 5)
30. Center at (0, 0); focus at (0, 1); vertex at (0, −2)
31. Foci at (±2, 0); length of the major axis is 6
32. Foci at (0, ±2); length of the major axis is 8
33. Focus at (−4, 0); vertices at (±5, 0)
34. Focus at (0, −4); vertices at (0, ±8)

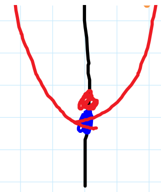
21) $$\frac{4x^2}{16}+\frac{y^2}{16}=\frac{16}{16}$$

$$\frac{x^2}{4}+\frac{y^2}{16}=1$$

$$\frac{x^2}{2^2}+\frac{y^2}{4^2}=1$$

cent: (0,0)





$$c^2 = 16 - 4 = 12$$

$$c = \sqrt{12} \approx 3.5$$

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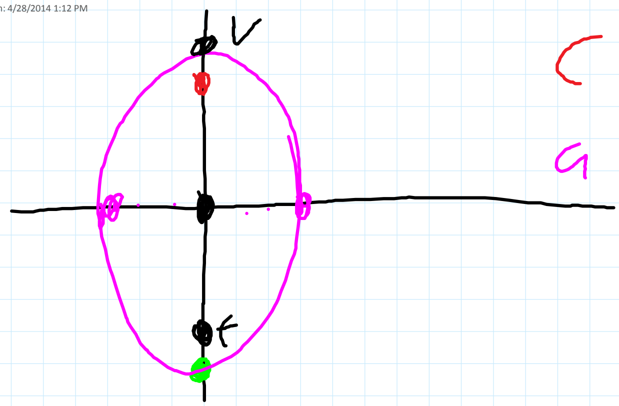
24)



$c = 4$

$a = 5$

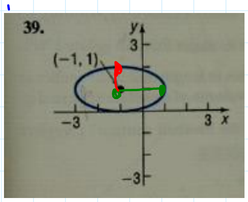
$c^2 = a^2 - b^2$

$4^2 = 5^2 - b^2$

$b^2 = 25 - 16 = 9$

$b = 3$

39)



C: $(-1, 1)$

a: 2

$b = 1$

$$\frac{(x+1)^2}{2^2} + \frac{(y-1)^2}{1^2} = 1$$

45) $$\frac{(x+5)^2}{16} + \frac{4(y-4)^2}{16} = \frac{16}{16}$$

$$\frac{(x+5)^2}{16} + \frac{(y-4)^2}{4} = 1$$

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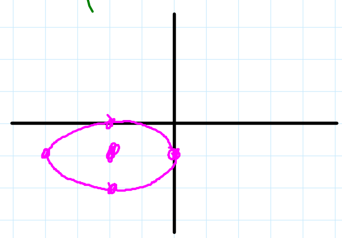
47) $x^2 + 4x + 4y^2 - 8y + 4 = 0$

$x^2 + 4x + 4 + 4y^2 - 8y = -4 + 4$

$x^2 + 4x + 4 + 4(y^2 - 2y + 1) = 0 + 4$

$(x+2)^2 + 4(y-1)^2 = 4$

$\frac{(x+2)^2}{4} + \frac{(y-1)}{1} = 1$ or $\frac{(x+2)^2}{2^2} + \frac{(y-1)^2}{1^2} = 1$



Center: $(-2, 1)$

$a = 2 \quad b = 1$

$c^2 = a^2 - b^2 = 3$

$c = \sqrt{3} \approx 1.7$

49) $2x^2 + 3y^2 - 8x + 6y + 5 = 0$

$(2x^2 - 8x) \; 3y^2 + 6y = -5$

$2(x^2 - 4x + 4) + 3(y^2 + 2y + 1) = -5 + 8 + 3$

$2(x-2)^2 + 3(y+1)^2 = 6$

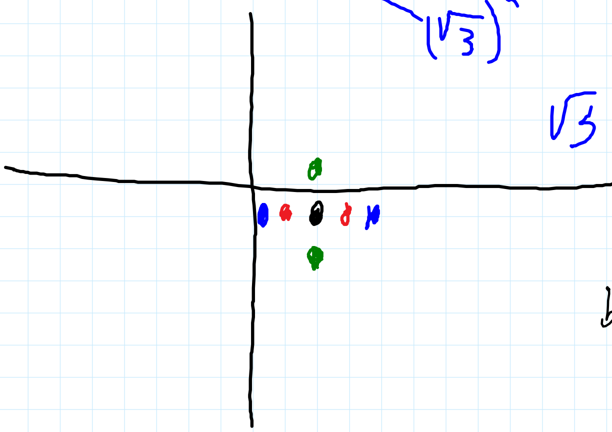
$$\frac{(x-2)^2}{3} + \frac{(y+1)^2}{2} = 1$$

$(\sqrt{3})^2$

$(\sqrt{2})^2$

$\sqrt{3} \approx 1.7$

$\sqrt{2} \approx 1.4$

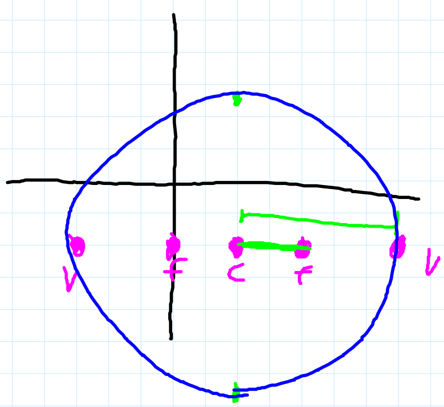


$b^2 = 21$

$b = \sqrt{21} \approx 4.5$

In Problems 55–64, find an equation for each ellipse. Graph the equation by hand.

55. Center at (2, −2); vertex at (7, −2); focus at (4, −2)   ~~56. Center at (−3, 1);~~ vertex at (−3, 3); focus at (−3, 0



$c^2 = a^2 - b^2$

$2^2 = 5^2 - b^2$

$b^2 = 25 - 4 = 21$

$b = \sqrt{21} \approx 4.6$