9.1 Sequences and Series

Sequence - List of numbers
Series - Sum of Sequence

Ex 1:

Write the first four terms of the sequence given by $a_n = 2n + 1$.

$a_1 = 2(1) + 1 = 3$ $a_4 = 9$

$a_2 = 2(2) + 1 = 5$

$a_3 = 2(3) + 1 = 7$

Ex 2:

Write the first four terms of the sequence given by $a_n = \frac{2 + (-1)^n}{n}$.

$a_1 = \frac{2 + (-1)^1}{1} = \frac{1}{1}$

$a_2 = \frac{2 + (-1)^2}{2} = \frac{3}{2}$

$a_3 = \frac{2 + (-1)^3}{3} = \frac{1}{3}$

$a_4 = \frac{2 + (-1)^4}{4} = \frac{3}{4}$

Ex 4:

Write the first five terms of the sequence defined recursively as

$\rightarrow a_1 = 6$

$\boxed{a_{k+1} = a_k} + 1$, where $k \geq 1$.

$a_2 = a_1 + 1 = 6 + 1 = 7$

$a_3 = a_2 + 1 = 7 + 1 = 8$

$a_4 = a_3 + 1 = 8 + 1 = 9$

$a_5 = 9 + 1 = 10$

a_3 a_2

a_{k+1} a_k

a_7 a_6

a_{k+1} a_k

a_k

Ex 5:

The Fibonacci sequence is defined recursively, as follows.

$$\begin{cases} a_0 = 1 \\ a_1 = 1 \end{cases}$$

$$a_k = a_{k-2} + a_{k-1}, \quad \text{where} \quad k \geq 2$$

Write the first six terms of this sequence.

$a_2 = a_0 + a_1 = 1 + 1 = 2$

$a_3 = a_1 + a_2 = 1 + 2 = 3$

$a_4 = a_2 + a_3 = 2 + 3 = 5$

$a_5 = a_3 + a_4 = 3 + 5 = 8$

$3! = 3 \; 2 \; 1$

$6! = 6 \; 5 \; 4 \; 3 \; 2 \; 1$

Definition of Factorial

If n is a positive integer, then ***n* factorial** is defined as

$$n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \cdot \cdot (n-1) \cdot n.$$

As a special case, zero factorial is defined as $0! = 1$.

Ex 6:

Write the first five terms of the sequence given by $a_n = \dfrac{3^n + 1}{n!}$. Begin with $n = 0$.

$a_0 = \dfrac{3^0 + 1}{0!} = \dfrac{2}{1}$

$a_1 = \dfrac{3^1 + 1}{1!} = \dfrac{4}{1}$

$a_2 = \dfrac{3^2 + 1}{2!} = \dfrac{10}{2} = \dfrac{5}{1}$

$a_3 = \dfrac{3^3 + 1}{3!} = \dfrac{28}{6} = \dfrac{14}{3}$

$a_4 = \dfrac{3^4 + 1}{4!} = \dfrac{82}{24} = \dfrac{41}{12}$

Definition of Summation Notation

The sum of the first n terms of a sequence is represented by

sigma $\rightarrow$ $$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \cdot \cdot \cdot + a_n$$

where i is the **index of summation,** n is the **upper limit of summation,** and 1 is the **lower limit of summation.**

Ex 7:

End

Find the sum $\sum_{i=1}^{4}(4i + 1)$.

$a_1 = 4(1) + 1 = 5$

$+ \ a_2 = 4(2) + 1 = 9$

$+ \ a_3 = 4(3) + 1 = 13$

$+ \ a_4 = 4(4) + 1 = 17$

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Definition of Series

Consider the infinite sequence $a_1, a_2, a_3, \ldots, a_i, \ldots$.

1. The sum of the first n terms of the sequence is called a **finite series** or the **nth partial sum** of the sequence and is denoted by

$$a_1 + a_2 + a_3 + \cdot \cdot \cdot + a_n = \sum_{i=1}^{n} a_i.$$

2. The sum of all the terms of the infinite sequence is called an **infinite series** and is denoted by

$$a_1 + a_2 + a_3 + \cdot \cdot \cdot + a_i + \cdot \cdot \cdot = \sum_{i=1}^{\infty} a_i.$$

Ex 9:

For the series

$$\sum_{i=1}^{\infty} \frac{3}{10^i} = 0.\overline{3} = \frac{1}{3}$$

find (a) the third partial sum

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$$\sum_{n=1}^{3} \frac{3}{10^n}$$

$a_1 + a_2 + a_3$

$.3 + .03 + .003 = .333$

9.1 pg 617: 7-25 odd, 59-73 odd, 89, 91

65) $\frac{(n+1)!}{n!}$ $\frac{6!}{5!} = \frac{6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$

$\frac{(n+1) \cdot n \cdot (n-1)(n-2) \quad 1}{n \cdot (n-1)(n-2) \cdot \quad 1}$

69) $\sum_{n=3}^{5} \frac{1}{n^2-3}$

$\frac{1}{3^2-3} + \frac{1}{4^2-3} + \frac{1}{5^2-3}$

89) $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$

a) $3^{rd} \rightarrow \sum_{n=1}^{3} \left(\frac{1}{2}\right)^n$

$$\left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$