Sequences and Series

Sequence - $\frac{L_{1} \leq T}{}$ of numbers Series - $\frac{Sum}{}$ of Sequence

Ex 1:

Write the first four terms of the sequence given by $a_n = 2n + 1$.

$$\Omega_1 = 2(1) + 1 = 3$$
 $\Omega_2 = 3(3) + 1 = 5$
 $\Omega_3 = 3(3) + 1 = 7$

Ex 2:

Write the first four terms of the sequence given by
$$a_n = \frac{2 + (-1)^n}{n}$$
.

$$C_1 = 2 + (-1) = \frac{1}{1}$$

$$C_2 = 2 + (-1)^2 = \frac{3}{2}$$

$$C_3 = 2 + (-1)^3 = \frac{3}{2}$$

$$C_4 = 2 + (-1)^3 = \frac{3}{2}$$

$$C_4 = 2 + (-1)^3 = \frac{3}{2}$$

Write the first five terms of the sequence defined recursively as

$$a_{1} = 6$$

$$a_{k+1} = a_{k} + 1, \text{ where } k \ge 1.$$

$$a_{1} = a_{1} + 1 = (+1 = 7)$$

$$a_{2} = a_{3} + 1 = 7 + 1 = 8$$

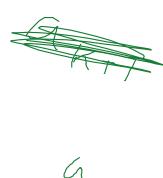
$$a_{4} = a_{3} + 1 = 8 + 1 = 9$$

$$a_{5} = a_{7} + 1 = 8 + 1 = 9$$

$$a_{k+1} = a_k$$

$$a_{k+1} = a_k$$

Ex 5:





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The Fibonacci sequence is defined recursively, as follows.

$$\begin{cases} a_0 = 1 \\ a_1 = 1 \end{cases}$$

$$a_k = a_{k-2} + a_{k-1}, \text{ where } k \ge 2$$

Write the first six terms of this sequence.

$$\Omega_{3} = \Omega_{0} + \Omega_{1} = 1 + 1 = 2$$

$$\Omega_{3} = \Omega_{1} + \Omega_{3} = 1 + 2 = 3$$

$$\Omega_{4} = \Omega_{3} + \Omega_{5} = 2 + 3 = 5$$

$$\Omega_{5} = \Omega_{3} + \Omega_{4} = 3 + 5 = 8$$

Definition of Factorial

If n is a positive integer, then n factorial is defined as

$$n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \cdot \cdot (n-1) \cdot n.$$

As a special case, zero factorial is defined as 0! = 1.

Ex 6:

Write the first five terms of the sequence given by
$$a_n = \frac{3^n + 1}{n!}$$
. Begin with $n = 0$.

$$0 = \frac{3^0 + 1}{0!} = \frac{3^n + 1}{n!}$$

$$0 = \frac{3^n + 1}{n!} = \frac{4}{1!}$$

$$aggle = \frac{3^3 + 1}{3!} = \frac{10}{3} = \frac{5}{1}$$

$$O_3 = \frac{3^3 + 1}{37} = \frac{28}{6} = \frac{14}{3}$$

$$C_{14} = \frac{3^{11} + 1}{4^{11}} = \frac{82}{3^{11}} = \frac{41}{13}$$

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Definition of Summation Notation

The sum of the first n terms of a sequence is represented by

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \dots + a_n$$

where i is the **index of summation**, n is the **upper limit of summation**, and 1 is the **lower limit of summation**.

Ex 7:

Find the sum
$$\sum_{i=1}^{4} (4i + 1)$$
. $G = G(1) + 1 = 5$
 $G = G(2) + 1 = G(3) + 1 = G(3)$

$$G_{1} = 4(1) + 1 = 5$$

$$+ \alpha_{1} = 4(3) + 1 = 9$$

$$+ \alpha_{3} = 4(3) + 1 = 13$$

$$+ \alpha_{4} = 4(4) + 1 = 17$$

$$\overline{44}$$

Definition of Series

Consider the infinite sequence $a_1, a_2, a_3, \ldots, a_i, \ldots$

1. The sum of the first *n* terms of the sequence is called a **finite series** or the *n*th partial sum of the sequence and is denoted by

$$a_1 + a_2 + a_3 + \cdots + a_n = \sum_{i=1}^n a_i$$

2. The sum of all the terms of the infinite sequence is called an **infinite series** and is denoted by

$$a_1 + a_2 + a_3 + \cdots + a_i + \cdots = \sum_{i=1}^{\infty} a_i$$

Ex 9:

For the series

$$\sum_{i=1}^{\infty} \frac{3}{10^i} = \sqrt{3} = \frac{1}{3}$$

find (a) the third partial sum

9.1 pg 617: 7-25 odd, 59-73 odd, 89, 91

$$\frac{(n+1)!}{n!} = \frac{(0) 5/0 1/1 3/1 1/1}{5!} = \frac{(0) 5/0 1/1 3/1 1/1}{(0) 1/1} = \frac{(0) 5/0 1/1 3/1 1/1}{(0) 1/1}$$

$$(9) \stackrel{5}{\underset{1=3}{\leq}} \stackrel{1}{\underset{1-3}{\vee}}$$

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$$(\frac{1}{3})^{1} + (\frac{1}{3})^{3} + (\frac{1}{3})^{3}$$
 $\frac{1}{3} + \frac{1}{4} + \frac{1}{8}$