9.2 Arithmetic Sequences and Partial Sums

Definition of Arithmetic Sequence

A sequence is **arithmetic** when the differences between consecutive terms are the same. So, the sequence

~~$a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$~~

is arithmetic when there is a number d such that

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots = d.$$

The number d is the **common difference** of the arithmetic sequence.

1, 2, 4, 8

The nth Term of an Arithmetic Sequence

The nth term of an arithmetic sequence has the form

$$a_n = a_1 + (n - 1)d$$

where d is the common difference between consecutive terms of the sequence and a_1 is the first term.

1, 3, 5, 7

2 2 2

Ex 2:

Find a formula for the nth term of the arithmetic sequence whose common difference is 5 and whose first term is -1.

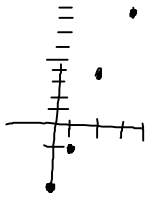
$a_n = a_1 + (n-1)d$

$a_n = -1 + (n-1)5$

$a_n = -1 + 5n - 5$

$a_n = 5n - 6$

3



Ex 3:

The 8th term of an arithmetic sequence is 25, and the 12th term is 41. Write the first 11 terms of this sequence.

8

$$d = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{41 - 25}{12 - 8} = 4$$

$a_n = a_1 + (n-1)d$ -3, 1, 5,

$25 = a_1 + (8-1)4$

$25 = a_1 + (7)4$

$-3 = a_1$

8 9 10 11 12

25 41

$41 = 25 + 4d$

$16 = 4d$

$4 = d$

Ex 4:

Find the ninth term of the arithmetic sequence whose first two terms are 2 and 9.

2 9

7

$a_n = a_1 + (n-1)d$

$a_9 = 2 + (9-1)7$

$a_9 = 2 + 56 = \boxed{58}$

The Sum of a Finite Arithmetic Sequence

The sum of a finite arithmetic sequence with n terms is given by $S_n = \frac{n}{2}(a_1 + a_n)$.

Ex 6

Find the sum of the integers (a) from 1 to 35

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{35} = \frac{35}{2}(1 + 35) = 630$

Ex 7

Ex 7

Find the 120th partial sum of the arithmetic sequence

6, 12, 18, 24, 30, ...

$a_n = a_1 + (n-1)d$

$a_n = 6 + (120-1)6$

$= 720$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{120} = \frac{120}{2}(6 + 720)$

$= 60(726)$

$= 43,560$

Ex 8

Find the 30th partial sum of the arithmetic sequence

78, 76, 74, 72, 70, ...

$a_n = a_1 + (n-1)d$

$= 78 + (30-1)(-2)$

$= 78 + 29(-2) = 20$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{30} = \frac{30}{2}(78 + 20)$

$= 15(98) = 1470$

9.2 pg 626: 5-17 odd, 23-53 odd

49. Sum of the first 100 positive odd integers

$a_1 = 1$

$a_{100} = a_1 + (n-1)d$

$a_{100} = 1 + (100-1)2$

$= 199$

arith $S_{100} = \frac{n}{2}(a_1 + a_n)$

$= \frac{100}{2}(1 + 199) = 10,000$