9.3 Geometric Sequences and Series

arith $a_n = a_1 + (n-1)d$

The *n*th Term of a Geometric Sequence

The nth term of a geometric sequence has the form

$$\boxed{a_n} = a_1 r^{n-1}$$

where r is the common ratio of consecutive terms of the sequence. So, every geometric sequence can be written in the form below.

$$\begin{array}{ccccccc} a_1, & a_2, & a_3, & a_4, & a_5, & \ldots, & a_n, \ldots \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \downarrow \\ a_1, & a_1 r, & a_1 r^2, & a_1 r^3, & a_1 r^4, & \ldots, & a_1 r^{n-1}, \ldots \end{array}$$

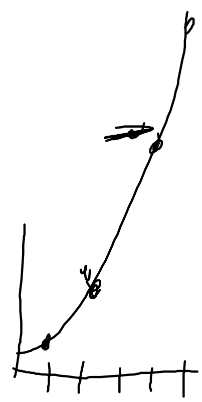
$a_n = a_1 \cdot r^{(n-1)}$

Ex 2.

Write the first five terms of the geometric sequence whose first term is $a_1 = 2$ and whose common ratio is $r = 4$. ~~Then graph the terms on a set of coordinate axes~~.

$2,\ 2 \cdot 4,\ 2 \cdot 4 \cdot 4,\ 2 \cdot 4^3,\ 2 \cdot 4^4$

$2,\ 8,\ 32,\ 128,\ 512$



Ex 4

$\frac{20}{4} = \frac{100}{20} = 5$

$a_n = a_1 \cdot r^{(n-1)}$

a) Find a formula for the nth term of the geometric sequence

4, 20, 100, ...

$a_n = 4 \cdot 5^{(n-1)}$

b) What is the 12th term of the sequence?

$a_n = 4 \cdot 5^{(12-1)} = 195{,}312{,}500$

Ex 5

The second term of a geometric sequence is 6, and the fifth term is 81/4. Find the eighth term. (Assume that the terms of the sequence are positive.)

$a_2,\ a_3,\ a_4,\ a_5$

The second term of a geometric sequence is 6, and the fifth term is 81/4. Find the eighth term. (Assume that the terms of the sequence are positive.)

a_2, a_3, a_4, a_5

$6 \qquad \frac{81}{4}$

$6 \cdot r \cdot r \cdot r = \frac{81}{4}$

$\frac{1}{6} 6 r^3 = \frac{81}{4} \frac{1}{6}$

$\sqrt[3]{r^3} = \sqrt[3]{\frac{81}{24}}$

$r = 1.5$

$\frac{a_2}{r} = a_1 = \frac{6}{1.5}$

$a_n = a_1 \cdot r^{(n-1)}$

$a_8 = 4 \cdot 1.5^{(8-1)}$

$= 68.343$

The Sum of a Finite Geometric Sequence

~~The sum of the finite geometric sequence~~

$\cancel{a_1, a_1 r, a_1 r^2, a_1 r^3, a_1 r^4, \ldots, a_1 r^{n-1}}$

with common ratio $r \neq 1$ is given by $S_n = \sum_{i=1}^{n} a_1 r^{i-1} = a_1\left(\frac{1 - r^n}{1 - r}\right)$.

Ex 6.

Find the sum $\sum_{i=1}^{10} 2(0.25)^{i-1}$.

$S_{10} = 2 \cdot \left(\frac{1 - (.25)^{10}}{1 - .25}\right) = 2.667$

The Sum of an Infinite Geometric Series

If $|r| < 1$, then the infinite geometric series

$$a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^{n-1} + \cdots$$

has the sum

$$S_\infty = \sum_{i=0}^{\infty} a_1 r^i = \frac{a_1}{1-r}.$$

Ex 7

Find each sum.

a. $\sum_{n=0}^{\infty} 5(0.5)^n$

$S_\infty = \frac{5}{1-.5} = \frac{5}{.5} = 10$

b. $5 + 1 + 0.2 + 0.04 + \cdots$

$a_1 =$ $r = \frac{1}{5} = \frac{.2}{1} = \frac{.04}{.2} = .2$

$S_\infty = \frac{a_1}{1-r} = \frac{5}{1-.2} = \boxed{6.25}$

9.3 pg 638: 5-15 odd, 23-27 odd, 33-41 odd, 47-50, 55-61 odd, 69, 71

59. $\sum_{n=0}^{20} 3\left(\frac{3}{2}\right)^n$ =

27, 54